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On the Structure Equation $F^4+F^3+F^2+F=0$

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Abstract

In this paper, we have studied various properties of F-sturcture satisfying $F^4+F^3+F^2+F=0$. Nijenhuis tensor and CR-structure have also been discussed.

Keywords: Differnetiable manifold, projection operators, Nijenhuis tensor and CR-structure.

Introduction

Let M^n be a C^∞ differentiable manifold and F be a (1,1) tensor of class C^∞, satisfying

$$F^4+F^3+F^2+F=0 \quad (1.1)$$

we define the operators *l* and *m* on M^n by

$$l=F^4, \qquad m=I-F^4 \quad (1.2)$$

From (1.1) and (1.2), we have

$$l+m=I, \quad l^2=l, \; m^2=m, \quad lm=ml=0 \quad (1.3)$$

$$Fl=lF=F, \quad Fm=mF=0.$$

Let

$$M=\{m-F^4,m-F^3,m-F^2,m-F,m+F,m+F^2,m+F^2,m+F^3,m+F^4\} \quad (1.4)$$

$$L=\{l-F^4,l-F^3,l-F^2,l-F,l+F,l+F^2,l+F^3,l+F^4,\} \quad (1.5)$$

We will study properties of some elements of *L* and *M*.

Theorem

Let us define

$$p=m+F^2, \quad q=m-F^2, \quad (1.6)\text{ i}$$

$$\alpha=m+F, \; \beta=m-F \quad \text{ii}$$

$$\gamma=l+F^2, \quad \delta=l-F^2 \quad \text{iii}$$

$$c=l+F, \quad d=l-F, \quad \text{iv, then}$$

$$pq=m-l, p^2=q^2=I, \; pl=-ql=F^2, pm=qm=m. \quad (1.7)$$

$$\alpha^2=\beta^2=p, \; \alpha\beta=q, \alpha^4=\beta^4=I, \alpha m=\beta m=m \quad (1.8)$$

$$\alpha l=-\beta l=F.$$

$$\gamma\delta=0, \quad \gamma^n=2^{n-1}\gamma, \; \delta^n=2^{n-1}\delta. \quad (1.9)$$

$$c^2+d^2=2\gamma, \quad cd=\delta \quad (1.10)$$

Proof

Using (1.2), (1.3) in (1.6), we get the results.

Nijenhuis Tensor

The Nijenhuis tensor *N* (*X,Y*) of *F* satisfying (1.1) is given by

$$N(X,Y)=[FX,FY]+F^2[X,Y]-F[FX,Y]-F[X,FY] \quad (2.1)$$

where the Lie bracket $[X,Y]$ is defined by

$$[X,Y]=XY-YX, \quad (2.2)$$ for arbitrary vector fields *X* and *Y* in M^n

Theorem (2.1)

For the F-structure satisfying (1.1) if

$$N(X,Y)=0 \cdot \text{ then} \quad (2.3)$$

$$F^3[[FX,FY]+F^2[X,Y]]=l[[FX,Y]+[X,FY]] \quad (2.4)$$

Proof

From (2.1) and (2.3), we get

$$[FX,FY]+F^2[X,Y]=F[[FX,Y]+[X,FY]] \quad (2.5)$$

Operaing by F^3 on both the sides of (2.5) and using (1.2), we get (2.4).

Theorem (2.3)

For the F-structure satisfying (1.1)

$$mN(X,Y)=m[FX,FY] \quad (2.6)$$

$$mN(F^3X,Y)=m[lX,FY] \quad (2.7)$$

Proof

From (1.3) and (2.1) we get (2.6). In (2.6) replacing X by F^3X and using (1.2), we get (2.7).

Cr-Structure

Let $T_C(M)$ be the complexified tangent bundle of M^n. A CR-structure on M^n is a complex sub-bundle H of $T_C(M)$, s.t. $H_p \cap \tilde{H}_p = 0$ and $P,Q \in H \Rightarrow [P,Q] \in H$. Here $\tilde{H}_p$ is c.c. of H_p. We define complex sub bundle H of $T_C(M)$ by

$$H_p = \{X - \sqrt{-1}FX,\ X \in X(D_l)\} \quad (3.1)$$

Where $X(D_l)$ denotes the $F(D_m)$ module of all differentiable sections of D_l. Here D_l and D_m are the distributons corresponding to the operators l and m respectively.

Theorem (3.1)

If $P,Q \in H,$ then

$$[P,Q]=[X,Y]-[FX,FY]-\sqrt{-1}(-1)[[FX,Y]+[X,FY]] \quad (3.2)$$

Proof

Let us define $P = X - \sqrt{-1}(-1)FX$ and $Q = Y - \sqrt{-1}(-1)FY$ and simplifying, we get (3.2).

Theorem (3.2)

The integrable F-structure satisfying (1.1) defines a CR-strucre F on it such that

3.3) $R_e(H)=D_e.$

Proof

Let $X,Y \in X(D_l),$ then $[X,FY],[FX,Y] \in X(D_L)$ using (1.3) and (2.3), we get

$$l[[X,FY]+[FX,Y]]=[X,FY]+[FX,Y] \quad (3.4)$$

Operating l on both sides of (3.2) and using (3.4) we get

$$l[P,Q]=[P,Q] \quad (3.5)$$

Consequently F satisfying (1.1) defines a CR-structure on Mn.

Definition (3.1)

Let $\tilde{K}$ be the complementary distribuition of $R_e(H)$ to $T(M)$, we define a morphism $F:T(M)\rightarrow T(M)$ by

$$FX = \begin{cases} 0 & if\ X \in X(\tilde{K}) \\ \frac{1}{2}\sqrt{-1}(-1)(P-\tilde{P}), & if\ X \in X(H_p) \end{cases} \quad (3.6)$$

where $P = X + \sqrt{-1}(-1)Y,$ and $\tilde{P}$ is c.c. of P from (3.6), we have

$$FX=-Y, F(-Y)=-X, \text{ or } \quad F^2X=-X \quad (3.7)$$

etc.

Theorem (3.3)

If Mn has a CR-structure H, then $F^4+F^3+F^2=F=0$ and consequently $R_e(H)=D_l$ and $\tilde{K}=D_m$.

Proof

Since F defines a CR-structure then from (3.7), $F^2X=-X,\ F^3X=-F(X),\ F^4X=X$

Thus

$$F^4X+F^3X+F^2X+FX=X-FX-X+FX=0 \quad (3.8)$$

and hence

$$F^4+F^3+F^2+F=0 \quad (3.9)$$

Aim of the Study

To establish that $F^4+F^3+F^2+F=0$ is a CR-structure.

Conclusion

$F^4+F^3+F^2+F=0$ is a CR-Structure.

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